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ANOMALOUS DIFFUSION IN REGULAR FRACTURED MEDIA

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Проанализирован процесс миграции примесей в гетерогенных средах, состоящих из низкопроницаемой матрицы с коэффициентом диффузии d, которая содержит узкие длинные области (“трещины”) с высокой диффузионной проницаемостью (коэффициент диффузии $D >> d$). Рассмотрены одиночные плоская и цилиндрическая трещины, системы из двух и из бесконечного числа параллельных трещин и “пересоединение” двух полубесконечных плоских трещин, концы которых подходят близко друг к другу. Для одиночной трещины показано, что в интервале времени $t_1 << t << t_2$ (где $t_1 = a^2 / 4d$, a – характерная толщина трещины, $t_2 = t_1(D / d)^2$ для плоских и $t_2 = t_1(D / d) \ln(D / d)$ для цилиндрических трещин) миграция примесей характеризуется субдиффузионным поведением с дисперсией $\sim D\sqrt{tt_1}$ в плоском и $\sim Dt_1 \ln (t / t_1)$ в цилиндрическом случае. Сходная ситуация возникает и во всех иных рассмотренных задачах: всегда существует интервал времен, при которых система находится в режиме субдиффузии, причем его длительность в большинстве случаев растет с D / d, так что, если D / d велико, его верхняя граница может оказаться практически недостижима. Показано, что “пересоединения” трещин не оказывают влияния на описанную картину смены режимов диффузии. Полученные результаты могут быть полезными в разработке методов оценок надежности захоронений радиоактивных отходов в скальных породах.

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Contaminant transport in regular heterogeneous media consisting of narrow areas (“fractures”) with diffusivity D and a matrix with the diffusivity d (D being much greater than d) has been analyzed. Considered are the following models: single flat and single cylindrical fracture, systems of two and of an infinite number of parallel fractures, and an “interconnection” of two semi-infinite flat parallel fractures, coming close to each other. For single fractures it has been found that in the time range of $t_1 << t << t_2$ (where $t_1 = a^2 / 4d$, a is a characteristic dimension of fracture cross-section, $t_2 = \left(D / d\right)^2 t_1$ for flat fractures and $t_2 = \left(D / d\right) \ln\left(D / d\right) t_1$ for cylinder ones) the contaminant transport is of subdiffusion behaviour with dispersion $\sim D\sqrt{tt_1}$ for flat and $\sim Dt_1 \ln\left(t / t_1\right)$ for cylindrical fracture; classical diffusion in the time ranges $t << t_1$ and $t >> t_2$ takes place with effective diffusivities D and d, respectively. Similar situation also arises for all considered fracture systems: there has always been found to exist a time interval (its duration being in the majority of cases monotonic with D / d), within which the system is in a subdiffusion mode. When the ratio D / d is extremely large (e.g., in case of rock fractures), the time t_2 might practically not be accessible resulting in that the regime of anomalous diffusion (subdiffusion) becomes to be asymptotic. Therefore, in connection with contaminant transport in fractured rocks, we may deal with the effect of suppression of the dispersion in fractures. Fracture “interconnections” are found to be inessential for complex systems diffusion time-patterns. The results obtained may be helpful for the development of methods to assess reliability of radioactive waste storage in rock massifs.

Anomalous Diffusion in Regular Fractured Media

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Over the last decades studies of contaminant migration processes in irregular media have experienced the period of intensive development (see, for example, [1] and references therein). In many respects, the reason is in the fact that, for most of the systems under consideration, time dependences of migrant dispersion prove to be anomalous with asymptotic power indexes other than one. Such systems cannot be described by the ordinary transport equation with regular and piecewise differentiable coordinate dependence of parameters, in other words, they have stochastic or fractal structure.

This paper is to analyze examples of simple systems, which – within an extremely wide time intervals covering many orders – reveal anomalous diffusion properties. The system architectures bear the marks of materials, which could be actually found in nature (or may be created), in particular, of various fractured rock.

The simplest model is a single "fracture" filled with a medium having the coefficient of diffusion* D (we may name it a "I" medium) and surrounded by an unlimited matrix (the medium "II") with the coefficient of diffusion d that is

$$D \gg d \tag{1}$$

More complex models assume the presence of two and infinite number of (parallel to each other) fractures governed by the inequality (1). It is of importance to consider a system simulating "interconnections" of fractures, i.e. the system where diffusion of two closely running but not contacting fractures occurs through a low permeable medium (often such interconnections determine behavior of a whole system, which is below the percolation threshold).

Our analysis will subject all such models.

1 Single Fracture

The Problem: Region I is limited in one or two dimensions and corresponds to the plane-parallel layer of thickness a or a straight cylinder (not mandatory circular) with the same diameter value. Region II covers the rest of the space.

At the initial moment of time, contaminant particles are concentrated in the region I, occupying a finite section with characteristic dimension $\leq a$. Our purpose is to find the time dependence of contaminant dispersion $<\vec{r}_t^{\,2}>$ defined by the relation:

$$<\vec{r}_t^{\,2}> = \frac{\int\limits_{(I)} d\vec{r}\ \vec{r}_t^{\,2}\, n(\vec{r},t)}{\int\limits_{(I)} d\vec{r}\, n(\vec{r},t)} \tag{2}$$

Here, $n(\vec{r},t)$ is contaminant concentration depending on spatial coordinates and time, $\vec{r}_t^{\,2} = (\vec{r}_t \cdot \vec{r}_t)$, where $\vec{r}_t$ is the projection of the radius vector $\vec{r}$ on the boundary plane if area I corresponds to a plane parallel layer (in this case $\vec{r}_t$ has dimensionality $p = 2$) or on the generatrix if region I is a straight cylinder (then the

* "Diffusion" does not necessarily mean "molecular diffusion". That may also be convection, and in this case the term "dispersion" is more frequently used.

dimensionality of $\vec{r}_t$ is $p=1$). An illustration of the type of media arrangement is presented on Fig. 1. The integrals in Eq. (2) are taken over the region *I*.

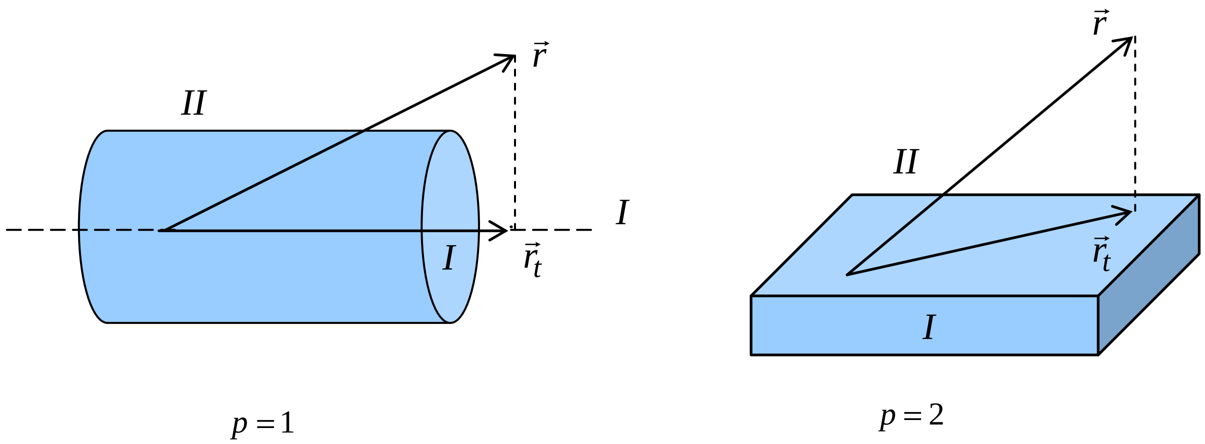

Fig. 1. *Geometry of the problem.*

1.1 Qualitative Analysis

In the time range $t << t_0$, where $t_0 = a^2/4D$ is the time of diffusion in medium I to a distance of order of its lateral dimensions, a, contaminant diffusion takes place within this medium as if it occupies the whole space. At the time $t >> t_0$, contaminant distribution in medium I becomes uniform along one (for plane parallel layer) or two (for cylinder) dimensions along which the medium is restricted. If additionally, the length of diffusion in environmental media is small compared with the lateral dimension of the medium I, i.e.

$$t << t_1, \qquad t_1 = a^2/4d \tag{3}$$

most of the time contaminant particles spend within the first medium, and we deal with diffusion that is quasi-two-dimensional at $p=2$ or quasi-one-dimensional at $p=1$. The contaminant dispersion in this case is given by the expression:

$$< \vec{r}_t^{\,2} > \sim 2p\,Dt, \qquad t_0 << t << t_1 \tag{4}$$

The situation changes for the time $t >> t_1$, when a fraction of time τ, which contaminant particles spend in medium I, becomes much less unity. The quantity τ by order of magnitude is determined by the ratio between volumes occupied by particles in media I and II:

$$\tau \sim \left(\frac{a}{\sqrt{dt}} \right)^{3-p} \tag{5}$$

Contaminant particle dispersion in medium I is estimated by the expression:

$$<\vec{r}_t^{\,2}> \sim D\int_{t_1}^{t} \tau(t')dt' \qquad (6)$$

Substituting Eq. (5) into Eq. (6), we find the following estimates:

$$<\vec{r}_t^{\,2}> \sim \frac{Da}{\sqrt{d}}\, t^{1/2} \qquad p = 2 \text{ (region } I \text{ is a plane-parallel layer)} \qquad (7)$$

$$<\vec{r}_t^{\,2}> \sim \frac{Da^2}{d}\ln\left(\frac{t}{t_1}\right) \qquad p = 1 \text{ (region } I \text{ is a straight cylinder)} \qquad (8)$$

The relations of Eq. (7) and (8) are correct under the condition that the dispersion in the medium *II* ($\sim d\,t$) remains to be much less the quantity $<\vec{r}_t^{\,2}>$. This corresponds to inequality $t << t_2$, where

$$t_2 \sim \left(\frac{D}{d}\right)^2 t_1 \qquad p = 2 \qquad (9)$$

$$t_2 \sim \frac{D}{d} t_1 \ln\left(\frac{D}{d}\right) \qquad p = 1 \qquad (10)$$

For the time $t >> t_2$, the dispersion $<\vec{r}_t^{\,2}>$ is completely determined by the diffusion law in the medium II, and we have

$$<\vec{r}_t^{\,2}> \sim 2p\,dt \qquad (11)$$

1.2 Problem Solution

The evolution of contaminant concentration is described by the ordinary diffusion equation that in medium I has the form

$$\frac{\partial n}{\partial t} = D\Delta n \qquad (12)$$

and in medium II is obtained from Eq. (12) by replacement $D \rightarrow d$. At times $t >> t_0$, when dispersion in the region I becomes to be much greater than the cross-section dimension of this region, a, contaminant distribution over this cross-section is uniform. Then, it is convenient to integrate Eq. (12) over the area of the cross-section and transform into Fourier representation on the coordinate $\vec{r}_t$ and Laplace representation on the time. After that, this equation takes the following form

$$\left(s + D\vec{k}^{\,2}\right) N_{\vec{k}\,s} + q_{\vec{k}\,s} = M \qquad (13)$$

Here, the following notations are used:

$$N_{\vec{k}\,s} \;=\; n_{\vec{k}\,s}\; S \tag{14}$$

$$n_{\vec{k}\,s} = \int_0^\infty dt \int d^{(p)}\vec{r}_t\; e^{-st - i\vec{k}\vec{r}_t}\; n(\vec{r}_t, t) \tag{15}$$

$q_{\vec{k}\,s}$ is Fourier-Laplace component of contaminant particle flux density, through the boundary between the regions I and II; S is cross section area at $p=1$ and $S=a$ at $p=2$; M is the total number of contaminant particles at initial time moment

$$M = \int d\vec{r}\; n(\vec{r}, 0) \tag{16}$$

In order to make the problem, concerning the medium I, to be closed, it is necessary to express the quantity $q_{\vec{k}\,s}$ through $N_{\vec{k}\,s}$. For this purpose, we use the diffusion equation in the region II, taking into account the continuity conditions for particle concentration and flux density at the boundary of two media. Let examine the cases of $p=2$ and $p=1$ separately.

A standard solution of diffusion problem in semi-infinite space with a specified flux density at a plane boundary results in the relation:

$$n_{\vec{k}\,s} \;=\; \int_{-\infty}^{\infty} \frac{d\kappa}{2\pi}\, \frac{q_{\vec{k}\,s}}{s \;+\; d\left(\vec{k}^{\,2} \;+\; \kappa^2\right)} \;=\; \frac{q_{\vec{k}\,s}}{2\sqrt{d\left(s \;+\; d\vec{k}^{\,2}\right)}} \qquad p=2 \tag{17}$$

Combining Eqs. (13), (14) and (17), we find expression for $N_{\vec{k}\,s}$:

$$N_{\vec{k}\,s} \;=\; \frac{M}{s \;+\; D\vec{k}^{\,2} \;+\; \sqrt{\dfrac{s \;+\; d\vec{k}^{\,2}}{t_1}}} \qquad p=2 \tag{18}$$

Note that this equality suits to describe contaminant concentration evolution for any time that meets condition $t >> t_0$.

To derive an equation of the Eq. (18) type when the region I has the form of a straight cylinder $(p=1)$, we require the fulfillment of stronger inequality: $t >> t_1$. In the case of an infinitely thin string the relation between n_{ks} and q_{ks} is

$$n_{ks} = \int_0^\infty \frac{\kappa \cdot d\kappa}{2\pi}\, \frac{q_{ks} \cdot J_0(\kappa r)}{s + d\left(k^2 + \vec{\kappa}^2\right)} = \frac{q_{ks}}{2\pi d} K_0\!\left(r\sqrt{\frac{s + dk^2}{d}} \right) \qquad p=1 \tag{19}$$

where J_ν and K_ν are Bessel and McDonald functions of the ν-th order respectively. When $t >> t_1$ $s << dk^2$, and the argument of K_0 for $r \cong a$ is much less than unity since the scale of any longitudinal non-uniformity $1/k >> a$. Therefore we may use the expansion (see, e.g., [2])

$$K_0(z) = -\ln\frac{z}{2} - C + O(z^2 \ln z) \qquad \text{when } z \to 0, \tag{20}$$

where $C \cong 0.577$ is Euler constant:

$$n_{ks} = \frac{q_{ks}}{2\pi\ d} \ln\left(\frac{2e^{-C}}{r\sqrt{k^2 + \frac{s}{d}}}\right) \qquad p=1 \tag{21}$$

Using the Gauss' theorem one can easily prove that in our limit (from $t >> t_1$ it follows that $\partial n / \partial t << D\Delta n$ and the theorem is applicable) this "string" solution satisfies uniform diffusion equation and the boundary condition for the arbitrary form of the cylinder cross-section. One can also see that the replacement of r on the boundary by its characteristic value $a = \sqrt{\frac{4S}{\pi}}$ doesn't affect the formula in the approximation used here. Thus, with account of Eq. (14), the solution of Eq. (13), as applied to times $t >> t_1$ for $p = 1$, takes the form:

$$N_{ks} = M\frac{t_1}{4}\ln\left(\frac{e^{-2C}}{st_1 + \left(\frac{ka}{2}\right)^2}\right) \qquad p=1 \tag{22}$$

Let proceed now to derive the contaminant dispersion defined by Eq. (2). Taking into account the homogeneity of concentration distribution over the cross-section of the region I at $t >> t_1$ and the formula for inverse Fourier-Laplace transformation

$$N(\vec{r},t) = M\int_c \frac{ds}{2\pi i}\int \frac{d^{(p)}\vec{k}}{(2\pi)^p}\exp\left(st + i\vec{k}\vec{r}\right)N_{\vec{k}s} \tag{23}$$

(the sign c at the integral denotes the Mellin contour), we obtain the following relation:

$$< \vec{r}_t^{\,2} > = \left[-\int_c \frac{ds}{2\pi i}e^{st}\left\{\frac{\partial^2}{\partial k_\alpha^2}N_{\vec{k}s}\right\}\Bigg|_{\vec{k}=0}\right]\left[\int_c \frac{ds}{2\pi i}e^{st}\, N_{0s}\right]^{-1} \tag{24}$$

Here, summation is over integer values of index α from 1 to p. Substituting the expressions of Eqs. (18), (22) into Eq. (24), we obtain contaminant dispersion at the times $t >> t_1$:

$$< \vec{r}_t^{\,2} > = 4\left(D\sqrt{\pi t t_1} + dt\right) \qquad p=2 \tag{25}$$

$$< \vec{r}_t^{\,2} > = 2Dt_1\ln\left(\frac{t}{t_1}\right) + 2dt \qquad p=1 \tag{26}$$

To describe the contaminant dispersion in the whole range $t >> t_0$, we propose interpolation formulas, which coincide asymptotically with Eq. (4) at $t_0 << t << t_1$ and with Eqs. (25), (26) at $t >> t_1$:

$$\left\langle \vec{r}_t^{\,2} \right\rangle = 2\pi D t_1 \left(\sqrt{1+\frac{4t}{\pi t_1}} - 1 \right) + 4dt \qquad p=2 \tag{27}$$

$$\left\langle \vec{r}_t^{\,2} \right\rangle = 2Dt_1 \ln\left(1+\frac{t}{t_1} \right) + 2dt \qquad p=1 \tag{28}$$

Fig. 2 shows schematically dispersion behavior defined by formulas (27) and (28).

A pattern of diffusion modes can be even more complicated, if the region I has a form, for example, of a straight cylinder with rectangular cross section having such sides a and b that $a^2/D >> b^2/d$. The following diffusion regimes will take place in this case:

1. Ordinary diffusion with diffusivity D at $t << b^2/d$
2. Square root time dependence of dispersion at $b^2/d << t << a^2/d$
3. Logarithmic time dependence of dispersion at

$$a^2/d << t << (D/d)(a^2/d)\ln(D/d)$$

4. Ordinary diffusion with diffusivity d at $t >> (D/d)(a^2/d)\ln(D/d)$.

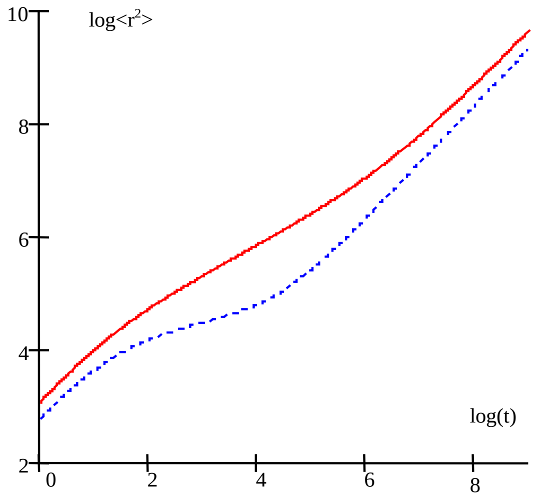

Fig. 2 *Log-log time dependence of the dispersion at $p=1$ (dash line) and at $p=2$ (solid line).*

2 Fracture Systems

Let us consider the systems consisting of two and of unlimited number of parallel flat fractures of a thickness, which are separated by a distance $b \gg a$. As before, at an initial point of time the diffusion agent is assumed to be concentrated in one of the fractures[†] in a small region of order a. The region covered by fractures will still be named "Region I" and the dispersion process will be governed by the formula (2).

At $t \ll t_b \equiv b^2/4d$ the contaminant does not "sense" the neighbouring fractures and diffusion goes in accordance with the dependencies obtained in the previous part.

But if $t \gg t_b$, both problems may be precisely solved by analytical means. In this case the concentration in any point of Region II is very easy to express through the flows at boundaries as it is done with the single fracture problem, and the exclusion of unknown values from the system of equations similar to (13) leads to the equations

$$\left[s + \left(d + \frac{2a}{b} D \right) k^2 + \sqrt{\frac{s + dk^2}{t_b}} \right] \sum_{j=1}^{2} N_{ks,j} = const \qquad (29)$$

in case of double fracture problem and

$$\left[s + \left(d + \frac{a}{b} D \right) k^2 \right] \sum_{j=-\infty}^{\infty} N_{ks,j} = const \qquad (30)$$

in case of unlimited fracture number problem. As a result for two fractures we obtain

$$\left\langle \vec{r}_t^{\,2} \right\rangle = 4\left[\left(d + \frac{2a}{b} D \right) \sqrt{\pi t_b t} + dt \right] \qquad (31)$$

and for the unlimited system

$$\left\langle \vec{r}_t^{\,2} \right\rangle = 4\left(d + \frac{a}{b} D \right) t \qquad (32)$$

A comparison with (25) leads to dependencies, which are qualitatively displayed on Figs. 3 and 4.

[†] It is easy to prove that such source position is not a loss of generality when we are interested in the migrant behavior within time intervals $t \gg t_b \equiv b^2/4d$. In case of time intervals $t \ll t_b$ fractures do not influence the migrant if a source is set a distance from each of them and dispersion is subject to the law $\left\langle \vec{r}_t^{\,2} \right\rangle \sim dt$.

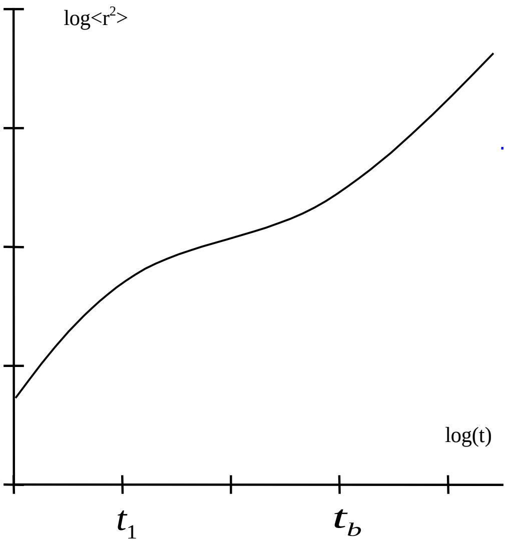

Fig. 3*a Scheme of log-log dispersion-time dependence for an infinite sequence of flat fractures at* $b \ll a\,\frac{D}{d}$ *(with asymptotic behaviour* $\sim \frac{a}{b}Dt$ *) and for a set of cylindrical parallel fractures, uniformly distributed in space, at* $b \ll a\sqrt{\frac{D}{d}}$ *(with asymptotic behaviour* $\sim \left(\frac{a}{b}\right)^2 Dt$ *)*

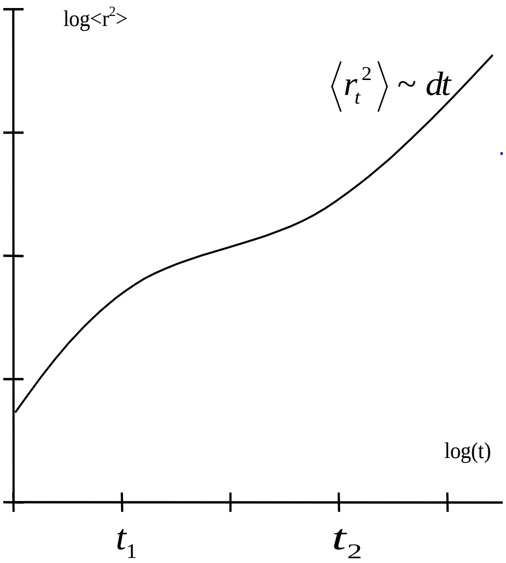

Fig. 3*b Scheme of log-log dispersion-time dependence for an infinite sequence of flat fractures at* $b \gg a\,\frac{D}{d}$ *and for a set of cylindrical parallel and uniformly distributed in space fractures at* $b \gg a\sqrt{\frac{D}{d}}$*. The scheme is also relevant to dispersion in a chain-like cylinder fracture system (Fig. 7) when* $b \gg a\sqrt{\frac{D}{d}}$ *.*

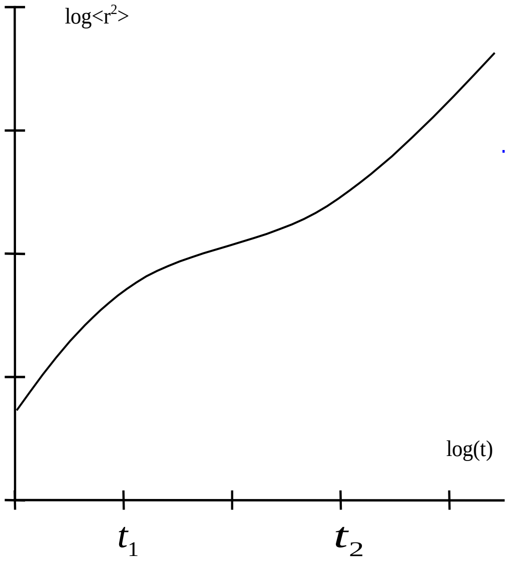

Fig. 4 *Scheme of log-log dispersion-time dependence for two parallel flat fractures and for two parallel cylindrical fractures in case* $b >> a\sqrt{\frac{D}{d}}$ *(in the opposite case* t_2 *must be replaced by* $\frac{D}{d}\frac{a^2}{d}\ln\frac{a}{b}\sqrt{\frac{D}{d}}$ *).*

As we see, solutions to the problems under question differ from that of the single fracture problem only in orders of typical time intervals and efficient coefficients of diffusion, but not in the appearance of time dependencies. In our view this is the evidence of non-sensitivity of the structure of diffusion behavior tending to a more complex fracture structure in the matrix. Another situation, however, could arise either in case of linkage type change (topology) in the highly permeable medium or in case of situations adjacent to these bifurcations where there are relatively narrow approaches (gaps) between unlinked fractures. Let us verify this possibility.

The simplest model containing such gaps is the problem of the instant source diffusion where the source contains M number of particles and located in point $x = -c$, and diffusion is to the medium shown in Fig. 5. Our interest is how the diffusion agent will be proliferating in Region 3, i.e. in the fracture, which is not contacting with the one containing the source (Region 1).

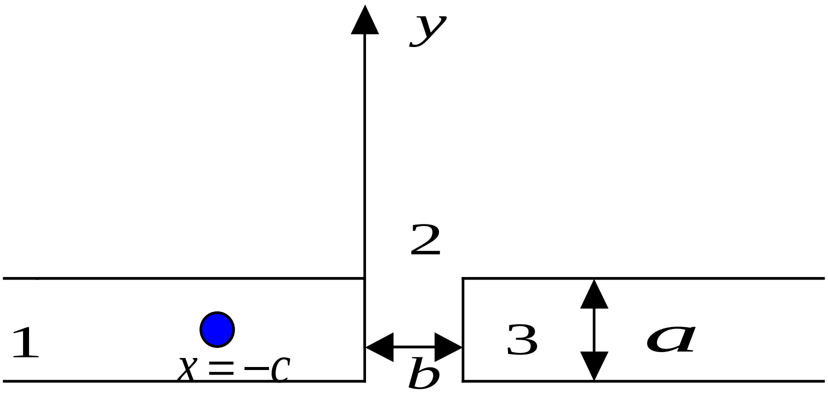

Fig. 5 *Scheme of an interconnection of fractures*

Firstly, assume that $b \ll a$. In this case, for the time intervals of order t_b, we may neglect the diffusion agent egress from a band with width a, the equations being considered one-dimensional. Matching of their solutions at $x = 0$ and $x = b$ produces

$$N_{1s} = \frac{M}{2\sqrt{sD}} \left(e^{-|x+c|\sqrt{\frac{s}{D}}} + \left\{ 1 - 2\frac{ch\left(b\sqrt{\frac{s}{d}}\right) + \sqrt{\frac{d}{D}}\, sh\left(b\sqrt{\frac{s}{d}}\right)}{\sqrt{\frac{D}{d}}\left(1 + \frac{d}{D}\right) sh\left(b\sqrt{\frac{s}{d}}\right) + 2\, ch\left(b\sqrt{\frac{s}{d}}\right)} \right\} e^{(x-c)\sqrt{\frac{s}{D}}} \right) \quad (33)$$

$$N_{3s} = \frac{M}{\sqrt{sD}} \frac{e^{-(x+c-b)\sqrt{\frac{s}{D}}}}{\sqrt{\frac{D}{d}}\left(1 + \frac{d}{D}\right) sh\left(b\sqrt{\frac{s}{d}}\right) + 2\, ch\left(b\sqrt{\frac{s}{d}}\right)} \quad (34)$$

where N_1 and N_3 are $\boldsymbol{N}$ in Regions 1 and 3, respectively.

We see that at $t \ll t_b$, i.e. $b\sqrt{\frac{s}{D}} \gg 1$, in Region 1 a normal diffusion in a semi-infinite segment with the zero flux boundary condition takes place, and in Region 3 concentration remains equal to zero. In the otherwise limiting case $t \gg t_b$ (34) is simplified:

$$N_{3s} = \frac{M}{\sqrt{sD}} \frac{e^{-(x+c)\sqrt{\frac{s}{D}}}}{2 + \frac{b}{d}\sqrt{sD}} \quad (35)$$

and we obtain

$$N_3 = \frac{M}{b}\frac{d}{D}\, e^{\frac{2d}{bD}(x+c) + \left(\frac{2d}{b}\right)^2 \frac{t}{D}}\, erfc\left(\frac{x+c}{2\sqrt{Dt}} + \frac{2d}{b}\sqrt{\frac{t}{D}}\right) \quad (36)$$

(see [3], p. 518).

It may be proven that the law of dispersion corresponding to (36) will never differ from the classical one. It is seen, however, that N_3 starts change substantially (as $M / \sqrt{Dt}$ does) only at time intervals $t \gg \frac{D}{d}\frac{b^2}{4d}$ (if, of course, $b \ll a\sqrt{\frac{d}{D}}$ and (36) remains true). We have to find out physical nature of this condition. It means that a flux involving a usual diffusion spreading and carrying particles from Region $x \cong 0$ surpasses the flux through the “hard” Region 2. Actually, the first flux $\boldsymbol{F_1}$, in terms of order of magnitude, is $\frac{\partial}{\partial t}\left(\Delta N\sqrt{Dt}\right)$, and the second one $F_2 = d\frac{\Delta N}{b}$ (herein is $\Delta N \equiv N_1 - N_3$); $F_1 \gg F_2$ actually means

$t >> \frac{D}{d}\frac{b^2}{4d}$. Having these time intervals the gap does not influence diffusion at all. (One has to be reminded that $t << \frac{a^2}{4d}$ should be true.)

Let us now proceed to the case $a << b$. Here the precise expressions are more difficult to obtain. However the resolved problems will be useful to carry out a meaningful qualitative analysis.

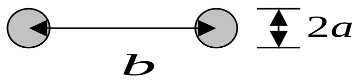

Fig. 6 Two cylindrical fractures

When $t << t_b$ Regions 1 and 3 are in no way interactive and the problem is similar to the one considered in the first part, which deals with two mutually mirror-like sources in points $x = \pm c$ (they ensure the zero flux condition at $x = 0$). That means that we will have $\langle \vec{r}_t^{\,2} \rangle \sim Dt$ at $\frac{a^2}{4D} << t << \frac{a^2}{4d}$ and $\langle \vec{r}_t^{\,2} \rangle \sim D\sqrt{\pi t t_1} + dt$ at $\frac{a^2}{4d} << t << \frac{b^2}{4d}$.

Consider the case $t >> t_b$. It should be noted that if $t_b >> t_2$, i.e. $b >> a\frac{D}{d}$, dispersion starts already by the time t_b according to the law $4dt$: the diffusion agent had departed the fracture and is so far away that it does not "sense" neither it nor its neighbour, and the second fracture, which is too remote from the first one, misses the game. Otherwise ($t_1 << t_b << t_2$), the contaminant approaches the second fracture by the time when the first fracture is in the subdiffusion mode and nearly all particles are in medium 2 to form a "blanket" of size $\sqrt{dt} >> a$. Therefore, the transfer through the part of Region 2, which meets the condition $|y| < a/2$, is $a/\sqrt{dt}$ times less efficient than through the remaining part of the Region, i.e. at $t >> t_b$ the exchange takes place mainly between the "blankets", the time of this exchange (t_b) being much less than the diffusion time along the system "fracture – blanket". In other words, at $t >> t_b$ a common "blanket" is formed at Regions 1 and 3 to ensure such efficient exchange that the system behaves as the one considered in the first part of this paper.

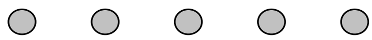

Fig. 7 A chain of cylindrical fractures

So, if $t_1 << t_b << t_2$, both intervals- $t_1 << t << t_b$ and $t_b << t << t_2$ feature subdiffusion; if, otherwise, $t_b >> t_2$, again at $t_1 << t << t_2$ subdiffusion takes place.

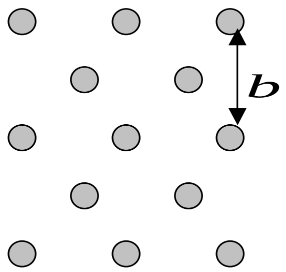

Fig. 8 Uniformly distributed cylindrical fractures

The last system we examine deals with cylindrical fractures, parallel and distanced by b from each other, as before. This model completes the symmetry between the first and the second part of the paper.

At first, let there be two fractures (a cross-section of them is depicted on Fig. 6). We could see that for $t >> t_b$ fractures are "unified", an efficient size being b and efficient diffusion coefficient being an average over this b-size region (i.e. about $d + \left(\frac{2a}{b}\right)^2 D$ in our case). So, as soon as for $t >> t_b$ the dispersion (perpendicular to the list plane) obeys (28), we meet two different situations: if $t_b >> t_2$, conventional diffusion ($\sim dt$) begins at t_2 and continues after t_b (Fig.4); if $t_b << t_2$, after t_b subdiffusion goes on, transforming into conventional mode at $t_1 \frac{D}{d} \ln \frac{a}{b} \sqrt{\frac{D}{d}}$.

Next, consider infinite sets of cylindrical fractures. In contrast to what we had above, here we can occupy by them either the whole space (Fig. 8) or only its subset (such as a narrow band depicted on Fig. 7). If the first possibility is realized, then to the moment t_b a conventional diffusion begins with the coefficient about $d + \left(\frac{2a}{b}\right)^2 D$ and, depending on the relation between t_b and t_2, we have the situations shown on Figs. 3.

Of the opposite cases we view only the one when the fractures are set to form a chain of negligible curvature (Fig. 7). If $t_b << t_2$, at $t >> t_b$ this chain behaves as a continuous plane (as the common blanket for all cylinders is formed), and a logarithmic plateau transforms into a square-root dispersion-time dependence, which retains till $\left(\frac{D}{d}\right)^2 t_1$, i.e. till "$t_2$ for plane". This intricate behaviour is sketched on Fig. 9. The case $t_b >> t_2$ refers us to Fig. 3b.

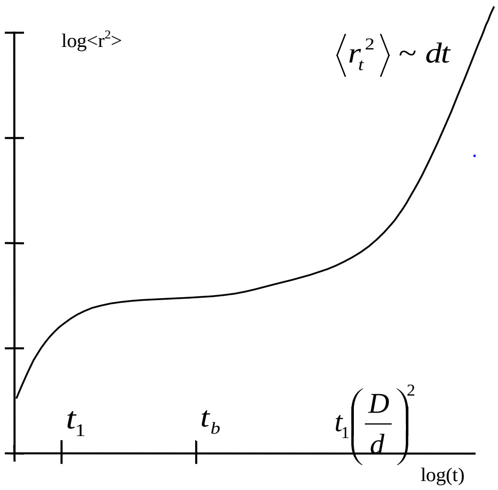

Fig. 9 *Scheme of log-log dispersion-time dependence for a chain-like cylinder fracture system (Fig. 7) when* $b << a\sqrt{\frac{D}{d}}$.

Conclusion.

By considering different examples we have got an evidence that in case of diffusion in the systems of "fractures" surrounded by a low permeable medium there is always a range of time intervals where dispersion grows slower with time than as per the linear law, however, surpassing dispersion in the hardly permeable medium. It should be especially stressed that at high D/d the upper boundary of the range may appear to be virtually inaccessible, and in this case, likewise irregular media, an off-normal mode would be viewed as asymptotic one.

An important conclusion that may be drawn from the work results is a low sensitiveness of diffusion mode change scheme to a complication of the fracture structure within the matrix. Moreover, the results are the evidence of the fact that the “interconnections” (regions where fractures are close to each other) are nearly irrelevant for the diffusion patterns. It is also worthwhile to note that for all considered fracture systems if the cease-time of subdiffusion t_2 exceeds the time of the diffusion through space between fractures t_b, the fractures behave collectively at $t >> t_b$. In the opposite case fractures act independently.

The problems resolved are of special interest in terms of radioactive waste disposal. Experimental studies of “tails” spreading from storage facilities differ by orders from theoretical assessments based on regular diffusion asymptotic forms. Fractured rock is the typical location of such storage facilities. Therefore, the studies of a particle diffusion behavior is important to understand processes leading to the above difference and to develop methods to evaluate reliability of disposal sites. The obtained results may be useful for these purposes.

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