Большов Л.А., Дыхне А.М., Кондратенко П.С. ФЛУКТУАЦИОННЫЕ АСПЕКТЫ ДИФФУЗИИ В СИЛЬНО НЕУПОРЯДОЧЕННЫХ СРЕДАХ. Препринт № IBRAE-2002-02. Москва: Институт проблем безопасного развития атомной энергетики РАН, 2002. 6 с.

Аннотация

Предложено обобщение модели транспорта примесей в сильно неупорядоченных средах, обладающих фрактальными свойствами, с учетом супердиффузионного поведения на больших расстояниях и флуктуационного – на малых расстояниях. Установлено, что пространственные флуктуации характеристик среды приводят к перенормировке мощности источника примесей. Коэффициент перенормировки, K, значительно убывает с размером источника R при значениях R меньше корреляционной длины, определяемой свойствами среды. В этой же области R коэффициент K, а с ним и эффективная мощность, испытывают возрастающий статистический разброс.

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Abstract

An extension of the model for an impurity transport in highly disordered media with fractal properties is proposed taking into account super-diffusion (at large distances) and fluctuation (at short distances) behavior. It is determined that spatial fluctuations of medium characteristics lead to renormalization of the power of an impurity source. Renormalization factor K considerably decreases with source size R if the latter is less than a correlation length determined by medium properties. In this range of R the factor K as well as an effective power of the source experience an increasing statistic scattering.

Fluctuation Aspects in Diffusion over Highly Disordered Media

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Impurity particle migration in highly disordered media with fractal properties is used to be analyzed basing on generalized transport equation which results in anomalous diffusion [1,2]. Such description is of averaged nature. However, it is obvious that local characteristics of fractal medium are highly fluctuating. The question arises: how do these fluctuations affect transport processes and how can relevant effects be taken into account? This paper is to analyze the impact of fluctuations on impurity transport processes depending on an impurity source size.

A generalized scheme of an averaged description of the impurity transport in statistically homogeneous media in three-dimension case can be formulated as a continuity equation:

$$\frac{\partial c}{\partial t} + div\vec{q} = 0 \tag{1}$$

Herein, the flux density vector $\vec{q} = \vec{q}(\vec{r}, t)$ and the particle concentration $c(\vec{r}, t)$ depending on coordinates and time are connected with an integral relation:

$$q_i(\vec{r}, t) = -\int d\vec{r}'\, f_i(\vec{r} - \vec{r}')\, c(\vec{r}', t) \tag{2}$$

The function $f_i(\vec{r})$ being determined by medium characteristics has the property:

$$\int d\vec{r}\, f_i(\vec{r}) = 0 \tag{3}$$

When this function $f_i(\vec{r})$ at long distances decreases faster than $|\vec{r}|^{-4}$, Eq. (1) is reduced to the classic diffusion equation resulting in that the migration length at a long time interval behaves as $r(t) \propto t^{1/2}$ and the concentration at long distances drops due to Gauss law. If the decrease of the function $f_i(\vec{r})$ is slower than $|\vec{r}|^{-4}$, this function, in view of the integral convergence in Eq. (2), has the following asymptotic:

$$f_i(\vec{r})\big|_{|\vec{r}|>>L} \cong d\left(\frac{\vec{r}}{|\vec{r}|}\right)\frac{V}{L^3}\left(\frac{L}{|\vec{r}|}\right)^{4-\alpha}, \qquad 0 < \alpha < 1 \tag{4}$$

Here, $d(\vec{n}) \sim 1$ if $|\vec{n}| = 1$ and V and L are medium characteristics having dimension of speed and length respectively. Substitution of Eq. (4) into Eq. (2) and then into Eq. (1) results in a generalized transport equation applicable for averaged description of impurity transfer if spatial scale of a variation in concentration exceeds a correlation length L, i.e. $L|\nabla c| << c$. In this case, transport processes are of super-diffusion nature with a migration length varying as $r(t) \sim L(Vt/L)^{1/(2-\alpha)}$. An alternative statement of this problem consists in the formalism of fractional derivatives. In this formulation, the impurity transport equation differs from the classic one by replacement of the spatial second-order derivatives by $(2-\alpha)$- order derivatives. It should be stressed that both formulations deal with averaged description of the processes at $|\vec{r}| >> L$ scales.

Consider the problem when the size of an impurity source R is of the order or less than the correlation length L, and suppose that the time counted from the beginning of source action satisfies the inequality: $t >> (L/V)$. Let us surround the source by an imaginary surface S_1 with characteristic radius $R_1 > L$.

The shape of this surface will be choused from the condition that the impurity concentration at the surface is to be constant for the point source located at the origin of coordinates being the center of the real source. A total impurity flux Q from the source under consideration to the surface S_1 can be represent in the form:

$$Q = A(c_0 - c_1) \tag{5}$$

Here c_0 and c_1 are the meanings of the impurity concentration at the source boundary surface S and at the surface S_1, respectively; the quantity A is determined by the medium properties in the space between the surfaces S and S_1 (in the near zone). The flux Q being continuous when crossing the surface S_1 is expressed through the medium characteristics outside the surface S_1 (in the far zone) where the generalized transport equation holds, and we have:

$$Q = Bc_1 \tag{6}$$

Excluding the concentration c_1 from Eqs. (5) and (6) we arrive to the relations:

$$Q = KQ_0, \qquad Q_0 = Bc_0, \qquad K = A(A+B)^{-1} \tag{7}$$

Under a given concentration at the source surface, the quantity Q_0 corresponds to the source power derived neglecting the fluctuations of medium characteristics, and the quantity Q is the effective source power renormalized by fluctuations, K is a power renormalization factor.

The coefficient B can be calculated by standard method based on Eqs. (1), (2) and (4). By the order of magnitude, it equals:

$$B \sim VL^2 \left(\frac{R_1}{L}\right)^{1+\alpha} \tag{8}$$

At distances $|\vec{r}| >> L$ the averaged concentration is expressed through the effective power Q regardless the source size. In particular, the Eq. (1) and the relations of Eqs. (2) and (4) result in the following expression for a far tail of the concentration distribution:

$$c(\vec{r},t) = Q\frac{t^2}{2}\frac{\partial f_i}{\partial x_i} \propto Q\frac{t^2}{|\vec{r}|^{5-\alpha}} \tag{9}$$

The coefficient A cannot be deduced from the averaged transfer equation since it depends on distribution details of medium characteristics in the near zone where they are subject to strong fluctuations. This situation reminds the problem associated with tunnel barrier conductivity having been studied by Raikh and Ruzin [3]. So we shall use the approach developed in Ref. [3]. In our case, the transfer coefficient A, similarly to conductivity in Ref. [3], is determined by sparse favorable configurations, named as “punctures” (e.g., fractures in the case of impurity transport in rock massifs). The contribution F into the transfer coefficient A coming from a specific puncture is statistically distributed in a broad interval of magnitudes and may be represented by the expression $F = F_0 \exp(-u)$ where u is auxiliary variable taking meanings between 0 and ∞. The puncture density referred to the unit area of the source boundary surface S, as in Ref. [3], can be determined as:

$$\rho(u) = (S_0)^{-1}\exp[-\Omega(u)] \tag{10}$$

where $S_0^{1/2}$ is a characteristic cross size of a puncture being negligible in comparison to the average distance between them, $\Omega(u)$ is a function with the properties: $\Omega(u) >> 1$, $\frac{\partial \Omega}{\partial u} < 0$, $\frac{\partial^2 \Omega}{\partial u^2} > 0$. The ensemble average value of the transfer coefficient A takes the form:

$$< A >= S \frac{F_0}{S_0} \int_0^{\infty} du \, e^{-u-\Omega(u)} \tag{11}$$

The integrand in Eq. (11) possesses a sharp peak. Thus, we have up to a pre-exponential factor:

$$< A > = S\, a\,, \qquad a \approx \frac{F_0}{S_0} \exp\left[-u_{opt} - \Omega_{opt}\right] \tag{12}$$

where a is a specific value of transfer coefficient which does not depend on source boundary surface area, $\Omega_{opt} = \Omega(u_{opt})$. The value $u = u_{opt}$ corresponding to optimal punctures is determined by the relation: $\left(\frac{\partial \Omega(u)}{\partial u}\right)_{u=u_{opt}} + 1 = 0$. The applicability condition for the result expressed by Eq. (12) consists in that the average quantity of optimal punctures at the source surface area should be large, $S\rho(u_{opt}) >> 1$, or:

$$S > S_* \,, \qquad S_* = S_0 \exp(\Omega_{opt}) \tag{13}$$

In the case $S < S_*$, when the average number of optimal punctures is less than unity, the averaged transfer coefficient is determined by integral from Eq. (11) in which the lower limit has the meaning $u = u_f$ corresponding to the punctures whose averaged number is of the order of unity, $S\,\rho(u_f) = 1$. Then up to a pre-exponential factor we have:

$$< A >= S_*\, a \, \exp\left[-\left(u_f - u_{op\,t}\right)\right] \qquad\qquad S < S_* \tag{14}$$

The quantity u_f, according to its definition and with account of Eqs. (10) and (13), satisfies the equation:

$$\frac{S}{S_*} \exp\left[\Omega_{opt} - \Omega(u_f)\right] = 1 \tag{15}$$

It should be noted that for large size sources (when $S > S_*$) the quantity $< A >$ decreases with decreasing of the surface area S proportionally to S itself, however if we deal with small sizes (when $S < S_*$) it decreases faster (according to Eqs. (14), (15)).

One more effect caused by fluctuations of disordered medium properties is an increase in statistic scattering of the transfer coefficient A, taking place when the source size decreases. Calculations similar to that ones done for tunnel barrier conductivity in Ref. [3] lead to the conclusion that the relative scattering $\Delta(A) =< (A - < A >)^2 >^{1/2} / < A >$ is small when $S > S_*$, it is compared to unity when $S < S_*$; and may exceed unity in the case $S << S_*$.

Taking into account the results for transfer coefficient A, the assessment of Eq. (10) and the relations of Eq. (9) we arrive to the conclusion that the power renormalization factor tends to unity when the source size is large ($K \cong 1$ if $S >> S_*$). For small meanings of the source size, the averaged value of the factor K is determined by the expression:

$$< K > \cong \frac{< A >}{B} << 1 \quad \text{at} \quad S << S_* \tag{16}$$

The behavior of the averaged renormalization factor $M(\nu) \equiv < K >$ on the dimensionless source size variable ν defined as

$$\nu = 1 + \frac{\ln(S / S_*)}{\Omega_{opt}} \tag{17}$$

is shown schematically in Fig. 1.

When the source surface area decreases ($S < S_*$) the statistical scattering of the renormalization factor ($\Delta(K)$) grows up similar to the transfer coefficient A. It is obvious that the characteristic surface area S_* separating two different impurity transfer regimes ($S > S_*$ when fluctuations are negligible and $S < S_*$ when fluctuations play a significant role) has the order of L^2.

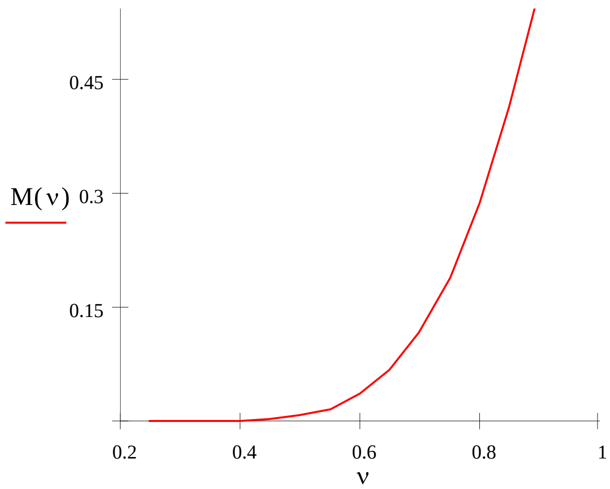

Fig. 1. Behavior of the averaged renormalization factor $M(\nu) \equiv < K >$ on the dimensionless source size variable ν defined by Eq. (17).

Thus, spatial fluctuations of medium characteristics result in significant reduction of the ensemble average effective power for the small size impurity source. In this case the effective power is subjected to a significant statistical scattering.

References

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